INTEGRAL REPRESENTATION OF ENERGY RELEASE RATE IN FUNCTIONALLY GRADED MATERIALS

Z.-H. Jin* and C. T. Sun**

* Department of Mechanical Engineering
The University of Maine
Orono, ME 04469, USA
zhihe.jin@maine.edu

** School of Aeronautics and Astronautics
Purdue University
West Lafayette, IN 47907, USA
sun@purdue.edu

For homogeneous elastic materials, Rice [1] introduced the path-independent *J* contour integral and derived it as the (potential) energy release rate. For general graded materials, the standard *J*-integral loses its path-independence and generally does not represent the energy release rate. Eischen [2] introduced a modification to the *J*-integral by adding an area integral term involving the explicit derivative of the strain energy density with respect to *x*, the coordinate along the crack direction and pointed out that the modified integral is the energy release rate based on a crack tip (local) field argument. This study presents a mathematically rigorous derivation of the modified *J*-integral [2] as the global (potential) energy release rate for graded elastic materials with continuous and piecewise differentiable properties.

Consider a two-dimensional nonhomogeneous body with a crack of length *a*. In the absence of body forces, the potential energy, Π, of the cracked body per unit thickness is a function of crack length *a* and can be expressed as

$$\Pi = \Pi(a) = \iint_{A_0} W dX dY - \int_{\Gamma_t} T_i u_i d\Gamma$$

where W is the strain energy density, T_i the prescribed boundary tractions on Γ_t, u_i the displacements corresponding to T_i on Γ_t, and A_0 the area of the cracked body. The energy release rate associated with a quasi-static crack extension is defined by

$$G = -\frac{d\Pi}{da} = -\frac{d}{da}\iint_{A_0} W dX dY + \frac{d}{da}\int_{\Gamma_t} T_i u_i d\Gamma$$

Following the approach of treating crack tip stress singularity in performing differentiation with respect to crack length *a* for homogeneous materials [3], we arrive at the following expression of energy release rate in general graded materials

$$G = J_{gm} = \int_{\Gamma}\left[W dy - T_i \frac{\partial u_i}{\partial x} d\Gamma \right] - \iint_{A_0} \frac{\partial W}{\partial x}\Big|_{\exp l}\, dx dy$$

which is the path/domain-independent J^*-integral (J_1^*) introduced in Ref. [2].

The modified *J*-integral is subsequently used to study the crack tip shielding and amplification due to a graded interlayer in an elastic-plastic material system.

References

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