STRESS CONCENTRATION AND LOCAL INSTABILITIES IN SOLIDS: SOME APPLICATIONS OF WKB TECHNIQUES

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Stress concentration in engineering structures such as beams, plates, and shells, subjected to various loading conditions, represents a frequent source of mechanical failure. In the most extreme cases such concentrations of stress lead to cracking and rupturing of the mechanical components, while in other instances they simply contribute to fatigue and a substantial reduction in the load-carrying capacity over prolonged periods of time. There is also an intermediate scenario, namely, when a structure subjected to *global* loading exhibits a *local instability* at sites within the structure that experience high levels of stress/strain. This aspect is related to *localised buckling*, a phenomenon which has received increasing attention over the past two decades or so (for example, see [1] and the references therein).

In this talk I shall discuss the application of WKB method to such problems, with a particular emphasis on the wrinkling of perforated thin elastic sheets in tension [2], or under torsional loading [3] (for a classical perspective on these problems, see [4]). In the absence of any bending rigidity the classical membrane theory leads to unsatisfactory results for such problems as it indicates the presence of a region of negative principal stress. Such inconsistencies are usually eliminated by adopting the framework of tension field theory, in which the stress field is not allowed to be compressive and wrinkling is assumed to occur in the direction perpendicular to the lines of uniaxial tension. The drawback of this approach is that it requires knowledge of the wrinkled region in advance, and no details about the local distribution of wrinkles are provided. To circumvent such difficulties, we allow for the presence of small stiffness in the elastic sheet and formulate the bifurcation equations as singularly perturbed fourth-order boundary value problems with variable coefficients. We show that the WKB method provides an effective tool for investigating these equations, and we study the dependence of the lowest eigenvalue on various non-dimensional quantities. Numerical results that back up our asymptotic results are included as well.

References

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