Traveling Wave and Rough Fault Earthquake Models: Illuminating the Relationship Between Slip Deficit and Event Frequency Statistics

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We discuss and interpret new simulation results from a recently proposed, physically-based model of earthquakes called the traveling wave model. This model produces a mixture of scaling and characteristic event ruptures. Stresses are transferred well beyond nearest neighbors in the two-dimensional lattice which represents the fault in the model. Cohesive forces due to small scale fault topography produce large scale friction, showing how friction is a function of length scale. Two variants of this new earthquake model, called 1) the basic traveling wave and 2) the rough fault model, illustrate the significant role the loading process has upon the statistics of the simulated events produced. Scale invariant rupture area distributions arise in these models in the same manner that they do in cellular automata slider block models, as a result of the force balance applied to the location of each site. Event frequency statistics are in the range of those observed for natural seismicity. Results from these models are compared to classical slider block models with long range springs.

1. INTRODUCTION

Models of earthquake sources can be broadly classified into three categories, observational, complex synthetic, and simplified synthetic. Models of specific earthquake observations generally focus on description of the patterns of moment release, not the source preparation process. [i.e. *Ruff*, 1992, *Mendoza et al.*, 1994.] By contrast, source preparation models have generally not been fit to observations of specific earthquake sequences. They are instead concerned with reproducing the generic properties of seismicity. Complex synthetic source preparation and rupture models include most known and empirically derived physical processes that operate in seismic source zones, and require strong material heterogeneity to produce populations of small earthquakes [*Rundle and Klein*, 1993, *Ben-Zion and Rice*, 1995]. Trade-offs and interactions between different physical processes can make interpretation of the results of the more complicated models difficult. These models often include long range stress transfer, dynamical formulations of the rupture process, explicit material heterogeneity and empirical friction relations, [*Mikumo and Miyatake*, 1979, *Ward*, 1992, *Ward and Goes*, 1993, *Rice* 1993, *Ben-Zion and Rice*, 1993].

Simplified synthetic models of source processes often neglect known physics in favor of easier interpretation and are better suited to finding the answers to thought experiments such as: Is long range stress transfer necessary for long range clustering? Is material heterogeneity necessary for slip complexity and the Gutenberg-Richter distribution? A large number of authors have constructed slider block [*Burridge and Knopoff* 1967] and other simplified source models. Examples include *Rundle and Klein* [1993], *Ding and Lu* [1993], *de Sousa Vieira et al.* [1993], *Lomitz-Adler* [1993], *Pepke et al.*, [1994] and *Rydelek and Sacks*, [1996].

One reason for the continued interest in slider block models is their ability to reproduce the power law distribution of earthquake magnitudes remarkably well, even though many of these models lack any effects of momentum, fault populations, or even explicit material heterogeneity. The dynamics of these models are expressed in the form of simple rules representing Coulomb friction and short range stress transfer. *Rundle and Kanamori* [1987] have shown that the healing process can produce heterogeneous stress distributions on faults that have failed in a large earthquake, and it is this healing process that perhaps most strongly distinguishes slider block models from more realistic models of earthquakes. The rapid healing causes slip to be concentrated in a pulse just behind the rupture front, another respect in which these models unexpectedly resemble the phenomenology of real earthquakes [*Heaton*, 1990, *Zheng et al.*, 1996].

The traveling wave model was first presented by *Rundle et al.* [1997] who also carried out an analysis of the model based upon statistical mechanics. The rough fault model presented below is a new variant of the traveling wave model. The present paper also presents some new interpretations of size distribution scalings. Both models are built on the tradition of simple models, but they represent an attempt to rationalize the evolution rules in terms of quantitative physical principles, so they are a step closer to more realistic source models. Rather than including detailed empirical descriptions of fault friction, we have instead taken the approach of seeing what friction properties our model can produce from the geometry of the rough fault in the model alone. Notice that we use the term "friction" here in the sense of a macroscopic resistance to sliding, in contrast to the cohesive forces over microscopic length scales on the surface. In the course of the work, we have gained a greater understanding of the mechanisms that produce the power law distribution of simulated earthquakes in all slider block models, and how these mechanisms are related to the configuration of the underlying slip deficit field. The strength of the coupling springs linking adjacent blocks has a strong influence upon the geometry of the slip deficit field, and also the size distribution of synthetic earthquakes. We have found two distinct scaling regimes corresponding to strongly and weakly coupled cases.

2. SIMULATION TECHNIQUES

The traveling wave and rough fault models presented here simulate earthquakes on a single two-dimensional fault with a square lattice of 10,000 sites distributed upon it. The dynamics are quasi-static, and there is no explicit material heterogeneity. As figure 1 shows, the sites rest upon a fault surface with a sinusoidal irregularity that moves relative to the loader plate, causing the loading springs to stretch as the sites move to follow the more energetically favored locations on the irregular fault surface. The same sinusoidal fault topography is experienced by each site, apart from variations in phase, so there is no spatial inhomogeneity of fault properties. Coupling springs connect not only the four nearest neighbors, as in most cellular automata slider block models, but also all other sites within r lattice units of each other, with coupling spring constants that decay with the distance cubed. The inverse-cube decay is characteristic of static stress transfer in three-dimensional elastic solids. The cases with $r = 2$ connect each site with 24 neighbors, and $r = 5$ connects each site with 120 neighbors. A model with range of interaction r connects each block to $(2r+1)^2 - 1$ neighbors. The equations describing temporal evolution of the basic traveling wave and rough fault models come from the minimization of the potential energy, which has a term that represents the effects of a rough surface, and that reproduces many of the effects of friction.

$\boxed{\text{figure 1}}$

The potential energy E is:

$$
\begin{aligned}
E &= \sum_{i=1}^{N} \frac{K_L}{2}\phi_i^2 \qquad (1)\\
&+ \sum_{j=-r}^{r} \frac{K_C}{2d(j)^3}(\phi_i - \phi_{i+j})^2 \\
&+ 2\gamma\cos(\kappa[\phi_i - vt + \epsilon_i]) + h\phi_i,
\end{aligned}
$$

in which the spring constant K_c controls the coupling between sites. Coupling stresses are transferred between sites a distance $d(j)$ apart following an inverse cube law out to the range of interaction r. The slip deficit $\phi = s_i(t) - vt$ is a measure of the displacement of each site behind the load point, whose loading spring has spring constant $K_L = 1$. The cosine term in the expression above can be thought of as the leading term in a Fourier expansion representing fault topography, with wavelength $2\pi/\kappa$, amplitude γ and phase that advances with velocity v as a function of time t. This cohesion term is a result of irregularities in the surface that create energetically favored locations for the sites. The variable ϵ_i is a phase scatter dependent upon location that is redistributed every time there is a major jump. Conceptually, every time a site moves past a feature on the fault surface, the next feature will appear at a location having a phase ϵ_i relative to the average position of features. The term $h\phi_i$ gives the potential well the simplest form of asymmetry, so it is included for generality. This expression can be solved to find the position of each site, ϕ_i, which minimizes the potential energy E as a function of the locations of the other sites and the coupling and loader spring constants, K_C and K_L.

[Figure 1 about here.]

The sketch in figure 1 shows the basic geometry of a one-dimensional slice through either the traveling wave or rough fault model. The cohesion term affects the force balance in the direction of site movement because normal stresses encourage the movement of the sites into the low spots, which are potential energy minima.

We find the net force on the i'th site by taking a negative derivative of the potential energy with respect to the slip deficit ϕ_i.

$$\begin{aligned} F_i &= K_L\phi_i \\ &+ \sum_{j=-r}^{r} \frac{K_C}{d(j)^3}(\phi_i - \phi_{i+j}) \\ &- 2\gamma\kappa\sin(\kappa[\phi_i - vt + \epsilon_i]) + h \end{aligned} \tag{2}$$

At each time t, the site locations ϕ_i are found that reduce the residual force F to a small value η. The level of accuracy, or threshold stress η can be thought of as representing the frictional effects of features smaller than the dominant wavelength $2\pi/\kappa$. The introduction of this threshold regularizes the problem and renders it much more tractable. Solutions have a tolerance or noise level η, because the locations of sites are only updated if their net forces are greater than η. Reducing the size of η allows us to approach the solutions which would be obtained in the case of a fault whose only friction arises from irregularities described by the potential in equation 1.

[Figure 2 about here.]

As an example of how the traveling wave model represents the earthquake process, consider figure 2, in which both the energy (1) and the force (2), are plotted for a single site and the noise $\epsilon_i = \eta_i = 0$. In the panels on the left, the term $-K_L\phi$ is the bold dotted line, and the solid and light dotted lines are the cosine terms at the two times t_0, t_1. The point(s) of intersection represent solutions to the equation $F(\phi) = 0$. The right panels show the corresponding points on the plot of $E(\phi(t))$. In the top panel, the quantity $\Lambda = 2\gamma\kappa^2/K_L$ is $\Lambda = 2$, whereas in the bottom panels, $\Lambda = 0.5$. As *Rundle et al.* [1996, 1998] have shown, when $\Lambda < 1$ (bottom), there is only one point of intersection, corresponding to the one stable solution. However, for $\Lambda > 1$, there can be one, two, or three points of intersection. A single solution point occurs when there exists only one minimum in the energy curve. Three solutions occur when there are two minima, separated by a maximum. Two solutions occur when the meta-stable minimum and the unstable maximum merge into one point.

figure 2

The distribution of the values ϵ_i of the phase scatter provides information about the spatial scale of the fault surface irregularities as compared to the lattice spacing. If the phase scatter is small, there is a strong correlation between the fault shape seen by all sites. Models of this kind might be suitable for studies of the preparation process, rupture and recovery of a large asperity, because these models contain events in which all sites jump simultaneously and quasi-periodically. The smaller events that occur in the adjustment process that follows a nucleation event of this type may resemble aftershocks.

The techniques needed to implement this model on a computer are different from conventional slider block models, because the usual "update rules" are replaced with a nonlinear routine based upon Newton's method for locating roots.

3. TEMPORAL EVOLUTION ALGORITHMS

3.1. *Basic Traveling Wave Model*

As described in *Rundle et al.* [1998], the basic equation to be solved for the slip deficit field $\phi_i(t)$ is based upon a gradient decent method subject to noise :

$$\frac{d\phi_i}{dt} = F_i + \eta. \tag{3}$$

However, due to the nonlinear nature of the sine term in F_i, this algorithm is computationally expensive to implement for large lattices. We have therefore been motivated to develop alternative approximate solution schemes that nonetheless yield the correct general temporal evolution and scaling behavior. These algorithms are described below.

1. Site locations ϕ and starting phases $\kappa\epsilon$ are either generated or read in from a prior model. Other parameters like the coupling spring constant K_C, the range of interaction r and the solution accuracy η are defined at this time as well.

2. The stresses F_i on all of the sites due to the cohesion, coupling and loading springs are computed.

3. The stresses on all the sites are compared with the solution accuracy η and the site closest to failure is selected. The failure time $t_f = t+\Delta t$ is found that makes the stress on that site exceed the threshold η. This time must be in the future, and is found by projecting the rate of change of F_i with slip deficit into the future,

$$\frac{\Delta F_i}{\delta\phi_i} \approx \frac{\partial F_i}{\partial\phi_i} = K_L - 2\gamma\kappa^2 \cos\kappa[\phi_i + vt + \epsilon_i]. \quad (4)$$

To estimate ΔF_i, we compare F_i to the small threshold η, hence

$$\Delta F_i \approx F_i - \eta. \quad (5)$$

Using $\delta\phi_i = -v\Delta t$, we finally have

$$\Delta t \approx \frac{\eta - F_i}{v\{K_L - 2\gamma\kappa^2\ \cos\kappa(\phi_i + vt + \epsilon_i)\}}. \quad (6)$$

4. The time is set to the estimated failure time, t_f. If the stress at that time, F_i is not greater than η, steps 2 and 3 are repeated. We increment the time only enough to bring the next site to failure and run the rupture without increasing time because of the tremendous difference between plate velocities, which move a few centimeters per year, and rupture velocities, which move a few kilometers per second.

5. Now the stress on the least-stable block is greater than the threshold, η. Before slip adjustment, the region of potential slip is set to the entire lattice.

6. Stresses are recomputed for the region of potential slip, and then compared with the threshold η. All those that exceed the threshold are flagged for adjustment. If there are no additional adjustments needed, statistics for this event are computed and saved, and the algorithm returns to step 3.

7. Using Newton's method, a new location for each site is calculated that satisfies

$$F_i(\phi_i) < \frac{\eta}{10}, \tag{7}$$

No sites are moved at this step.

8. The locations of the sites are then updated together. If the number of updates exceeds 20, the new location is averaged with the old one so that oscillations are damped. The region of potential slip is adjusted to contain all sites that slipped plus a buffer of width r, and then step 6 begins again.

3.2. Rough Fault Model

The evolution algorithm for the rough fault version of the traveling wave model is like the basic traveling wave model except for the loading process. During loading in the basic traveling wave model the sites change position relative to the cohesive forces. Loading in the rough fault model is different, because during loading the sites move to follow the features of the cohesion term, accumulating stress from the loading springs until they jump. Only steps 3 and 4 are different. To estimate the failure time in this case, we calculate Δt using the relation:

$$\frac{dF_i}{dt} \approx \frac{\Delta F_i}{\Delta t}. \tag{8}$$

We again estimate ΔF_i from (5). To calculate $\frac{dF_i}{dt}$, we differentiate (2):

$$\begin{aligned}
\frac{dF_i}{dt} &= \frac{d}{dt} F_i(\phi_j(t), t) = \Sigma_j \frac{\partial F}{\partial \phi_j} \frac{\partial \phi_j}{\partial t} + \partial F_i / \partial t \\
&= \{K_L - 2\gamma\kappa^2 \cos\kappa[\phi_i + vt + \epsilon_i]\}(-v) \\
&- \left(2\gamma\kappa^2 \cos\kappa[\phi_i + vt + \epsilon_i]\right) v \\
&= -K_L v
\end{aligned} \tag{9}$$

Combining (5), (8), and (9), we obtain

$$\Delta t \approx (\eta - F_i)/vK_L. \tag{10}$$

Step four computes the failure time from: $t_f = t + \Delta t$, which is then used to update the locations:

$$\phi(t + \Delta t) = \phi(t) + v\Delta t \tag{11}$$

and the stresses $F_i(t + \Delta t) = F_i(t) + vK_L\Delta t$.

When sites move in the rough fault model, they always release elastic energy. In the traveling wave model, sites must move to store elastic energy as well as to release it. The rules for slip adjustment are the same.

For the rough fault model, the solution accuracy η may be interpreted as a friction resulting from irregularities small compared to the microscopic length scale that characterizes the sites.

4. MODEL PHENOMENA

4.1. Friction Results

[Figure 3 about here.]

One of the most interesting properties of the traveling wave and rough fault models is their ability to model bulk, macroscopic friction, arising from "microscopic" cosine bumps. The cohesion term in the potential representing fault topography produces a bulk frictional force when it is averaged over the fault surface. This synthetic bulk frictional force resists the motion of the loader plate in whatever direction the loading force is applied, just as natural frictional forces do. Unlike conventional slider block models, the failure stress η does not dominate resistance to loader plate movement. Physically, the frictional force in the traveling wave and rough fault models arises as a result of the normal stress σ_N pushing the fault together, acting on the fault topography, that gives rise to the cohesive energy 2γ and h. This normal stress renders some displacements more energetically favored than others. The amplitude of the cohesion term γ is proportional to the normal stress σ_N. The displacement of a site thus represents a balance between coupling forces, loading shear stresses, small scale friction, and the cohesive energy (potential field) created by compressive stresses normal to the fault.

The small-scale friction term, or error term η is the frictional slip threshold that applies to single sites. The cohesive bulk frictional force is much larger scale, because it is an average property of the entire model, representing the average resistance to sliding of the whole system of 10,000 sites. Even when the small scale friction is made exceedingly small, the large scale friction remains essentially unchanged, because the cohesion comes from the balance between fault topography and normal stress. The topography given to the model fault in this case is a simple sinusoid, but it is still capable of producing cohesive forces which oppose the motion of the loading plate. The cohesive forces do not average to zero because the most energetically favored locations for sites are not symmetrically distributed about the minima in cohesive energy. Stable locations for sites are offset toward the loading direction from the cohesive energy minimum, which causes the cohesive force to oppose the motion of the loading plate. As figure 3 illustrates, cohesive forces that give rise to bulk friction $\boxed{\text{figure 3}}$

in the model are proportional to the coupling spring constant. That is, there is a significant dependence of the bulk friction on microscopic interactions between sites.

If the fault model is re-scaled, so that all the sites together represent a single block of a larger model, then bulk friction of the smaller scale becomes the sliding stress η of the large scale. This picture justifies the introduction of significant microscopic friction into models which are supposed to represent macroscopic fault scales. Faults possess irregularities on a broad range of scales, so it is reasonable to suppose that realistic models of faults might have significant frictional forces on multiple scales, as the models with nonzero η and cohesion do.

There are significant differences between the cohesive friction of the traveling wave models and classic Coulomb friction. The cohesive models have no explicit difference between static and dynamic macroscopic friction, because there are no processes modeled which are inherently time dependent.

4.2. Scaling Properties

The traveling wave and rough fault models produce sequences of failures that resemble natural earthquakes in their power-law size distributions and in some cases show increased proportions of large magnitude "characteristic" events. The mechanisms that produce the power-law distribution of small events are the same in the traveling wave and rough fault models as in classic slider block models, so much of the discussion that follows applies to all three types of models.

[Figure 4 about here.]

[Figure 5 about here.]

Sizes of simulated earthquakes in a slider block model are governed by the slip deficit distribution, also called the displacement field. Each site is displaced from its equilibrium position by an amount ϕ which is a function of location across the lattice. If the rupture front encounters a set of sites which are already displaced in the direction in which the rupture is causing sites to slip, the rupture may stop because the sites on the edge cannot be induced to fail by the stresses transfered from the rupture. Discontinuities in the displacement field correspond to bands of strong positive and negative stresses, features unfavorable for the propagation of ruptures. This means that discontinuities in displacement field are likely to arrest ruptures. If we call each region of similar displacement in the displacement field a segment, we expect the population of segments in the displacement field will at least in part control the population of simulated earthquakes generated. Relatively smooth fields with typically large distances between displacement discontinuities should produce simulations rich in large earthquakes. Rough distributions with short distances between discontinuities should produce more small area ruptures.

The examples shown in figure 4 are statistics from earthquake models of three different types and a few choices of model parameters. At small values of the noise threshold η, the cohesion term is very important, and the phase scatter ϵ strongly influences the scaling. Large amounts of phase scatter produce greater numbers of small area ruptures, and small phase scatter reduces the influence of the cohesion term significantly, producing distributions more similar to the original slider block model and to natural earthquakes. In effect, larger phase scatter (annealed disorder) serves to reduce the correlation length of the slip deficit field, thus producing smaller events. When η is very large, the cohesion term is only a small perturbation upon it, and all three types of models exhibit similar scaling. This scaling is dependent only upon the degree of coupling, K_c/K_L in the models.

figure 4

Variations in the range of interaction r do not produce any obvious systematic trends in the event-size statistics. Longer ranges of interaction do enhance the edge effects in the models though, and they increase the initial transient, meaning that many more events need to be simulated before the statistical characteristics stabilize. Longer ranges of interaction are also more realistic, since the range of interaction in the earth is effectively infinite, but the strongest stress transfer is at close ranges, and so we have usually used a range of interaction near 5, which includes interactions only 1% the size of the nearest neighbor stress transfer.

4.2.1. Power Spectra. In recent work *Rundle et al.*, 1998], John Rundle proposed a stochastic fracture mechanics framework that extends the usual Gibbs-Griffith approach to brittle shear fracture. This framework is an attempt to quantify the physical idea that a shear fracture will only extend through that region of a material in which the average difference between fracture strength $\sigma^F(x)$ and shear stress $\sigma(x)$ is sufficiently small. Thus we are interested in the statistical correlation properties of the field $\Sigma(x) = \sigma^F(x) - \sigma(x)$. In the slider block models of figure 4, $\sigma^F(x)$ is a spatial constant. For the traveling wave models, the analogous quantities, 2γ and h, are constants. Therefore, correlations in $\sigma(x)$ determine the correlations in $\Sigma(x)$, which result from curvature in $\phi(x,t)$, the slip deficit field (see equation 2).

Correlations in the slip deficit field can be analyzed by studying the power spectrum of slip deficit, which is the Fourier transform of the pair correlation function of the slip deficit between sites. Examples of power spectra of slip deficit fields for the models shown previously in figure 4 are given in figure 5 using methods that have been discussed extensively in the literature [*Voss*, 1988]. These power spectra were measured from simulation results by taking the slip deficit field $\phi(x,y,t)$, and extracting strips $\phi(x,t)$. These one dimensional data were then Fourier transformed and squared to produce the power spectra. The slopes of these power spectra at the high frequency end (large wavenumber k) yield values for the power spectrum exponent $-\beta$.

figure 5

Exponents β range from roughly zero to about 2. The latter value is also characteristic of brownian noise, as can be seen from the crosses (+). The theory of stochastic fracture [*Rundle et al.*, 1998] predicts that the eventual size of shear fractures should depend on the value of the Hurst exponent H characterizing correlations in slip deficit field. For $H = 0.5$, events of all sizes are expected. For values of H less than 0.5, system-wide nucleation ("characteristic") events are possible. Finally, for values of H greater than 0.5, a maximum size event is predicted.

We determined the value of H by measuring the slope of the power spectrum and using the relationship (*Voss*, [1988], and Appendix A):

$$D = E + 1 - H = E + \frac{3-\beta}{2} \tag{12}$$

In (12), E is the Euclidean dimension of the independent variable over which the power spectrum is computed. Because we compute the power spectrum for one dimensional curves, here $E = 1$. D is the associated fractal dimension of the curve, H is the Hurst exponent and β is the exponent characterizing the power spectrum. Note that equation (12) assumes that the curves are self similar over all scales, and thus, for example, that there is a well defined value for β, over all spatial frequencies, leading to well defined values of H and D. The power spectra for the slip deficits in figure 5 can be seen to have fixed slopes ($-\beta$) only at the high frequency (shorter wavelength) end of the spectrum in general. This implies the existence of a finite correlation length.

With these considerations in mind, we turn to an examination of figures 4 and 5. To check the predictions of stochastic fracture mechanics, we compare models that a) have a large enough value of r that stress concentrates at the rupture tip as the rupture grows, and b) preferably have the same value for K_C. The two models that best fit these criteria are the wave model with $K_C = 8$ and $r = 8$ (diamonds), and the slider block model with $K_C = 8$ and $r = 5$ (squares). Also plotted on figure 5 are power spectra for a brownian walk (crosses, $\beta = 2$, $H = 0.5$) and for white noise (dots, $\beta = 0$, $H = -0.5$). Figure 5 shows that the slider block model has a power spectrum that is very similar to the brown walk, except for the lowest frequencies, indicating a finite correlation length of perhaps 40 lattice units. By contrast, the wave model has a spectrum that corresponds to white noise out to the longest wavelengths, where the simulation data are less reliable. Comparing figures 4 and 5, we see that the slider block model with $H = 0.5$ has a frequency of occurrence curve (figure 4) that is linear out to the largest events as predicted. By contrast, the wave model with $H = -0.5$ has small and intermediate events, but an excess of large events.

4.2.2. Fractal Dimension. The fractal dimension D of the power spectrum is a function of the slope β (12). Setting $\beta = 2$ for the brownian case gives $D = 3/2$. The value of $\beta = 0$ for white noise yields a fractal dimension $D = 5/2$, also in rough agreement with the exponent from the distribution of areas. Notice that these fractal dimensions are the same as the values for the Fisher exponents τ in the frequency size distributions shown in figure 4. Our simulations suggest that physically, the coincidence between the values of D and τ arises because the rupture process detects features of the slip deficit field in much the same way as a box-counting algorithm determines fractal dimension. In fact, this physical idea can be regarded as an alternative statement of the fundamental principle underlying stochastic fracture mechanics.

Evidently, rupture propagation as defined in slider block models is sensitive to the fractal dimension of the underlying displacement field. Since ruptures are more likely to stop at discontinuities in displacement, the population of rupture areas is a measure of the population of displacement features in the displacement field on a fault. Thus, similarities in values of D and τ should not be completely unexpected.

4.2.3. Discussion. We interpret the scaling limits in terms of interactions between the model evolution and the displacement distribution. In the limit of weak coupling, the sites move independently. Each one has an equal chance to be in each stress state, because they independently cycle through all stress states, and the initial stress distribution is random. The uniform distribution of stresses maps to an approximately uniform distribution of displacements in simulations with weak coupling, because there is little stress transfer. A slider block model with weak coupling naturally produces a displacement field similar to white noise because the positions of neighboring blocks are nearly independent of one another. White-noise scaling is also found in cases of the traveling wave and rough fault model with very large phase scatter, because the site locations become uncorrelated.

For strongly coupled slider block models, stress transfer becomes the dominant contributor to the total stress exerted on each block. Blocks still have a uniform probability of being at any stress level over time, but this no longer maps to a uniform probability of being at any displacement level. Instead, it is gradients of the displacement field that generate stresses. The displacement accumulating across a lattice takes a step, positive or negative, depending upon the value of the stress on a particular block. The accumulation of slip deficit across a slider block lattice is similar to the accumulation of steps in a random walk, and so it is not surprising that it produces a population of ruptures having the same scaling properties as brownian noise.

If the slider blocks are assumed to be representative of parts of a flat fault, elasticity theory may be used to define what coupling spring constant and range of interaction are most physical. For adjacent cracks connected to one another by a three-dimensional continuous elastic medium, the displacement on one crack changes the stress on a neighboring crack 40% as much as it changes the stress on itself. This corresponds to a K_c/K_L ratio of 0.4. If the cracks nearest a slipped crack do not move, cracks two crack dimensions distant will still be loaded to some extent, but only 0.6% as strongly as the crack unloads itself. This argument provides some physical basis for neglecting the longest range stress transfer in a slider block model. If the cracks are not adjacent, being separated by unbroken material, the coupling will be reduced, and K_c/K_L ratios less than 0.4 become appropriate. Physically realistic cases with $K_c/K_L = 0.4$ exhibit scaling very similar to the cases with $K_c/K_L = 8$ shown in figure 4. Thus, the most physically reasonable choice for the coupling spring constants leads to physically reasonable distributions of event areas, similar to those observed for natural earthquakes. The models produce magnitude distributions that approximate observed Gutenberg-Richter magnitude distributions, with simulated b-values ≈ 0.85, slightly lower than the most typical values for real earthquakes. Since these models represent single homogeneous faults, it is not surprising that their statistics are not identical to b-values observed in entire networks of natural faults. Smaller events should be more numerous in networks of faults, where geometric and frictional irregularities produce effectively lower coupling and introduce barriers to rupture.

5. CONCLUSIONS

We have discussed some new results arising from the recently proposed traveling wave model for earthquakes, together with a new variation, the rough fault model. We presented evidence illustrating the relationship between the evolving slip deficit field and the frequency-area statistics for event occurrence. The basic result is that the strength of coupling between sites affects the statistics of the slip deficit field, which in turn determines the frequency-area relationship. For strong coupling (large K_c), one obtains brown noise statistics in the slip deficit, and for weak coupling (small K_c), the slip deficit field has statistics similar to white noise. These statistics determine both the power spectral exponent β through the relationship (12), and evidently also the value of the Fisher exponent τ, and ultimately the b-value of simulated events.

In addition to this basic result, we also find: (1) The large scale friction caused by the cohesion term in these models is proportional to K_c, the parameter defining the strength of coupling on the microscopic scale. The bulk friction is also proportional to normal stresses and oppose the direction of applied loading, like real friction. (2) Both the rough fault and basic traveling wave model produce characteristic events in the limit of low phase scatter and friction. (3) Power-law event statistics similar to earthquake size statistics arise in the traveling wave and rough fault models when frictional forces and stress transfer are significant compared to cohesion, as in a traditional slider block model.

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APPENDIX A: FRACTAL RELATIONSHIPS

To derive the relationship between the correlation of a random walk $V(x)$ and its fractal dimension D, [*Voss*, 1988, section 1.6.5] we first need to consider the statistical definition of a random walk with Hurst parameter H,

$$< |V(x+dx) - V(x)|^2 > \propto |dx|^{2H}. \tag{A1}$$

The definition of the autocorrelation function G is,

$$< |V(x+dx) - V(x)|^2 >= 2[< V^2 > -G_V(dx)], \tag{A2}$$

and the definition of the autocorrelation slope is

$$G_V \propto dx^{\beta-1} \tag{A3}$$

from which we can see that

$$2H = \beta - 1. \tag{A4}$$

Now it is necessary to relate the random walk parameter H to the fractal dimension D of the surface that includes the walk. Two dimensional slider block models are concerned with the displacement of blocks which occupy a two dimensional lattice. The slip deficit is the analog of V in the above equations, and the displacements accumulate in the spatial dimensions x and y instead of the more frequently encountered time. The three dimensions that describe the state of the lattice are two of space and one of slip deficit, but we are concerned with a single profile of slip deficit across the lattice, so only one of the spatial dimensions need be considered. Counting the number N of boxes needed to cover the set as a function of the length of a box L,

$$N \propto L^{-D} \tag{A5}$$

defines the fractal dimension D.

The boxes fill the spatial dimension x completely, because there is some slip deficit defined for every slider block. The only complication comes when we consider how many boxes are occupied by the lattice slice in the slip deficit dimension V. If the slice has the statistical properties of a random walk, it will occupy a number of boxes in the slip deficit dimension N_v which is related to the size of the boxes in the spatial dimension, L_x,

$$N_V = \frac{L_x^H}{L_V}. \tag{A6}$$

And the total number of occupied boxes in a slice of length X will scale like

$$N = \frac{X}{L_x} \cdot \frac{L_x^H}{L_V}. \tag{A7}$$

So if we reduce the two dimensions of the boxes together, making $L_x = L_V$, then the total number of boxes is

$$N = \frac{X}{L^{2-H}}, \tag{A8}$$

which implies

$$D = 2 - H. \tag{A9}$$

Combining equation 16 and 21 gives

$$D = 2 - \frac{\beta - 1}{2}, \tag{A10}$$

which may be solved for β,

$$\beta = 5 - 2D. \tag{A11}$$

Acknowledgments. I appreciate the constructive comments made by anonymous reviewers. Financial support of S.J.G. was provided by NSF grant EAR-9526814. This work would not have been possible without extensive conversations and advice from John Rundle.

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